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Estimating the Solar Meridional Circulation by Normal Mode Decomposition

Lars Krieger[1], **Markus Roth**[2,1], and **Oskar von der Lühe**[1]

[1] Kiepenheuer-Institut für Sonnenphysik, Schöneckstraße 6, 79104 Freiburg, Germany
[2] Max-Planck-Institut für Sonnensystemforschung, Max-Planck-Straße 2, 37191 Katlenburg-Lindau, Germany

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The objective of this article is to use Fourier-Hankel decomposition as suggested earlier by Braun & Fan (1998) to estimate the integrated horizontal meridional flow velocity as a function of mode penetration depth, and to find ways of potentially improve this technique. We use a time series of 43200 (30 days) consecutive full-disk Dopplergrams obtained by the MDI (Michelson Doppler Imager) instrument aboard the SOHO (Solar Heliospheric Observatory) spacecraft in April 1999. We find averaged meridional flow estimates of 15 m/s for modes with a penetration depth in the upper 20 Mm of the solar convection zone. This reproduces the results of the earlier investigations. Moreover we conclude that this method has the potential to become a new technique to measure the meridional circulation in the deep convection zone, if some improvements will be applied.

1 Introduction

On the solar surface a meridional flow of 10–20 m/s can be measured. This flow is directed away from the equator towards the poles (Hathaway (1996), Komm, Howard & Harvey (1993), Latushko (1996), Snodgrass & Daily (1996)). Because of mass conservation a poleward meridional flow in the outer part of the convection zone requires an equatorward return flow in deeper layers. This meridional circulation might play an important role in the dynamo process which causes the solar magnetic cycle (Choudhuri, Schüssler & Dikpati (1995), Dikpati & Gilman (2006), Dikpati & Charbonneau (1999), Brandenburg, Moss & Tuominen (1992)). It has been the topic of various studies (starting with Giles et al. (1997)) to infer the depth-dependent profile of this meridional circulation with various techniques from helioseismology.

The motivation for this work is to reproduce a method suggested by Braun & Fan (1998) for inferring the speed of the averaged solar meridional flow. The method is based on splitting up the oscillation signal observed in a time series of Dopplergrams into two wave-fields of anti-parallel travelling waves by a Fourier-Hankel decomposition (Braun, Duvall, & Labonte (1987)). Originally an average meridional flow in the order of 10 m/s was found in the upper part of the convection zone. We are able to reproduce the results on an independent data set and with a newly created data analysis pipeline. The reproduction of the method provides a first step in measuring the meridional circulation. We discuss potential improvements of this method to obtain more accurate results and to possibly reach to greater depths in future studies.

2 Methods and data

2.1 Normal mode decomposition

Braun, Duvall, & Labonte (1987) applied Fourier-Hankel spectral analysis to decompose locally the solar oscillation signal into wave fields travelling in opposite directions. An overview to the technique is given in Gizon & Birch (2005). This decomposition was used for investigating the influence of sunspots on the helioseismic p-modes. It allowed detecting phase-shifts and damping of p-modes by sunspots (Braun, Duvall & Labonte (1988), Bogdan et al. (1998), Braun (1995)).

A further application allowed studying the effect of the meridional flow on the solar oscillations (Braun & Fan (1998)). However, the Fourier-Hankel decomposition was originally designed for studying signals on a small section of a spherical surface by approximating the surface by a planar geometry. Therefore it is generally not suited for studying global flows on a sphere. Nevertheless, we reproduce the data analysis in the following. But we note, this method can only be seen as an initial approximative step to obtain rough estimates of average effects. It might allow to conclude on what to expect from a more proper analysis in future.

The velocity amplitude signal of the solar eigenoscillations $\Psi(\theta, \varphi, t)$, consisting of standing p-modes, can be locally separated into two fields of travelling waves on the sphere. This local decomposition is carried out within an annular region around a central point of interest. This point is the piercing point of the axis of the used spherical coordinate system, where θ is the latitude, and φ is the longitude. In the original concept (Braun, Duvall, & Labonte (1987))

this point has been the center of a sunspot or an active region. In the following we investigate a subsection of an annular region around the poles of the sun, i.e. the polar axis of the coordinate system is identical with the rotation axis of the sun. The coordinates used therefore coincide with the standard spherical coordinate system.

According to Braun & Fan (1998) the oscillation velocity signal Ψ is expanded as

$$\Psi(\theta,\varphi,t) \;=\; \sum_{lm\nu} e^{\mathrm{i}(\mathrm{m}\varphi+2\pi\nu\mathrm{t})} \times \left[A_{lm\nu}\Theta_l^m(\cos\theta) + B_{lm\nu}(\Theta_l^m)^*(\cos\theta)\right] \;, \tag{1}$$

with

$$\Theta_l^m(\cos\theta) := N_l^m \left[P_l^m(\cos\theta) + \frac{2\mathrm{i}}{\pi} Q_l^m(\cos\theta)\right] \;, \tag{2}$$

where the asterisk denotes complex conjugation, P_l^m and Q_l^m are Legendre functions of the first and second kind, and N_l^m is a normalisation constant. The complex quantities $A_{lm\nu}$ and $B_{lm\nu}$ are the amplitudes of the inward and outward going waves respectively, n is the radial order, l is the harmonic degree, m is the azimuthal order, t is time and ν is the temporal frequency. We refer the reader to Braun & Fan (1998) and Braun, Duvall & Labonte (1988) for more details on these decomposition.

Following Braun & Fan (1998) Hankel functions are used as approximations to $(P_l^m \pm 2\mathrm{i}/\pi \mathrm{Q_l^m})$

$$H_m^{(1,2)}(L\theta) \;\approx\; (-1)^m \frac{(l-m)!}{(l+m)!} \times \left[P_l^m(\cos\theta) \pm \frac{2\mathrm{i}}{\pi} Q_l^m(\cos\theta)\right] \;, \tag{3}$$

with $L = \sqrt{l(l+1)}$. This approximation is valid in the limit $l \gg m$, which is fulfilled in the current investigation (Braun (1995)). A decomposition into wavefields by applying a method built on Legendre functions, rather than Hankel functions, would be more accurate for the spherical case. But the application of Hankel functions provides an estimation which is much faster to calculate.

According to Gizon & Birch (2005) in some cases a discrete set of Hankel functions can be selected which guarantees orthogonality. Explicitly an integration over the entire range of φ would provide such an orthogonal set of functions in the present case. In the further data analysis, however, the integral is carried out only over a limited range in φ. This then introduces possibly leakage effects between modes of different azimuthal order m. Therefore only the best possible approximation to an orthogonal set can be actually selected.

The wave amplitudes $A_{lm\nu}$ and $B_{lm\nu}$ are extracted from the wave field $\Psi(\theta,\varphi,t)$ according to Braun et al. (1992) by

$$A_{lm\nu} \simeq C \int \Psi(\theta,\varphi,t) H^{(2)}(L_j\theta) \times e^{-(\mathrm{i}m\varphi+2\pi\nu t)}\, \theta \,\mathrm{d}\theta \,\mathrm{d}\varphi \,\mathrm{d}t \;, \tag{4}$$

$$B_{lm\nu} \simeq C \int \Psi(\theta,\varphi,t) H^{(1)}(L_j\theta) \times e^{-(\mathrm{i}m\varphi+2\pi\nu t)}\, \theta \,\mathrm{d}\theta \,\mathrm{d}\varphi \,\mathrm{d}t \;, \tag{5}$$

where C is a normalisation which is given approximately by $L/(4T\Theta)$ with Θ the range in latitude over which the integration is carried out, and T the duration of the observation.

Summarized, the signal is decomposed by a spatial Hankel transformation and a temporal Fourier transformation. The result of this decomposition is a set of frequency spectra $|A_{lm\nu}|^2$ and $|B_{lm\nu}|^2$ depending on the harmonic degree l. Peaks occurring in these spectra are to be identified as modes of different radial order n with the frequencies ν_{nl}^{pol} and ν_{nl}^{eq} respectively.

Potentially differences between the mode frequencies for poleward and equatorward propagating waves with same indices emerge due to advection. The frequency differences carry information about the influence of the horizontal component of the meridional circulation on the p-modes. Thus an exploration of the set of spectra leads to a set of frequency shifts $\Delta\nu_{nl} := \nu_{nl}^{\mathrm{pol}} - \nu_{nl}^{\mathrm{eq}}$ assigned to a respective set of unique indices.

2.2 Effect of meridional circulation on the p-modes

In the following we provide a formula that describes the effect of a horizontal flow $\mathbf{U}$ on the p-mode frequencies. For detailed steps to obtain this formula see Braun & Fan (1998) and references therein.

Under the influence of a horizontal flow $\mathbf{U}$ the net frequency shift between waves propagating poleward and equatorward is given by

$$\Delta\nu_{nl} = \frac{l \int_0^{R_\odot} (\langle U\rangle_\theta / r) K_{nl}(r)\,\mathrm{d}r}{\pi \int_0^{R_\odot} K_{nl}(r)\,\mathrm{d}r} \;, \tag{6}$$

where $K_{nl}(r)$ is a depth- and mode-dependent weighting function and

$$\langle U\rangle_\theta := \frac{1}{\theta_{\mathrm{max}} - \theta_{\mathrm{min}}} \int_{\theta_{\mathrm{min}}}^{\theta_{\mathrm{max}}} \mathbf{U}\cdot\hat{\boldsymbol{\theta}}\,\mathrm{d}\theta \;. \tag{7}$$

is the flow average over the spanned polar angle. Following Braun & Fan (1998) the frequency shift of a zonal mode due to advection is proportional to the horizontal wavenumber and the quantity

$$\langle U\rangle := \Delta\nu_{ln}\pi R_\odot / l \;, \tag{8}$$

gives the weighted depth average of the angular mean meridional flow $\langle U\rangle_\theta / r$ multiplied by the solar radius.

Together with

$$\bar{\nu} := \frac{1}{2}\left(\nu_{nl}^{\mathrm{pol}} + \nu_{nl}^{\mathrm{eq}}\right) \;, \tag{9}$$

all these considerations yield a data set $\{(\langle U\rangle, \bar{\nu}/L)_i\}$, where each i corresponds to a given p-mode.

In the last step the turning points of the respective modes are determined by using the relation

$$\frac{c^2(r_t)}{r_t^2} = \frac{(2\pi\bar{\nu})^2}{l(l+1)} \;, \tag{10}$$

with r_t the depth of the inner turning point. From this we obtain the mode penetration depth $\epsilon := R_\odot - r_t$ below the solar surface.

We note that the estimation of the integral carried out is valid only in very shallow layers as it required approximating $\langle U \rangle_\theta / r$ by $\langle U \rangle_\theta / R_\odot$. The velocity amplitude inferred later is therefore only reliable within the outer layers of the Sun. In deeper layers, however, this estimation might only provide information about the quality of the velocity profile, i.e. the sign of the weighted velocity average.

2.3 Data analysis

The analysed data consist of a time series of full disk Dopplergrams recorded by SOHO/MDI. The timeseries lasts from April 1–30, 1999 and consists of 43200 consecutive single Dopplergrams. Each Dopplergram is given on a 1024×1024 pixel grid. The duty cycle of the time series is better than 99%. The data used by Braun & Fan (1998) were timeseries of MDI and GONG Dopplergrams from 1997.

Each Dopplergram is projected onto an equidistant θ-φ-lattice, referring to the standard three-dimensional spherical coordinate system, θ-centered at the solar north pole. After the transformation the Dopplergram is asymmetric with respect to the equator, because at the observing time the northern hemisphere of the Sun was better visible. The oscillation signal is therefore better measurable on the northern hemisphere.

Two areas on the solar surface are investigated, one in the northern the other in the southern hemisphere. The two investigated zones are placed symmetrically to the equator with a polar interval of $\theta_{\text{tot}} = \pi/4$ and an azimuthal interval of $\varphi_{\text{tot}} = \pi/2$. The selected areas are centered at $\{\theta = \pi/4, \varphi = \pi/2\}$ and $\{\theta = 3/4\,\pi, \varphi = \pi/2\}$ respectively. The sizes of the fields were chosen such to yield the best oscillation signal-to-noise ratio. Moreover this allows the resolution of oscillations with low harmonic degree l in order to reach to greater depths in the solar interior. This choice is comparable with the investigation carried out by Braun & Fan (1998).

The geometry of the chosen fields restricts the possible resolution of the harmonic degree to $l \geq 4$ as a lower boundary. The resolution of the Dopplergrams yields an upper boundary of $l \leq 1024$. As we assume a φ-independent meridional circulation, only modes with $m = 0$ are investigated.

3 Results

3.1 General results for both hemispheres

The result of our investigation are measurements of the average horizontal meridional velocity $\langle U \rangle$ as a function of ν/L. Besides the relation $\langle U \rangle \sim \nu/L$ we are interested in the average horizontal velocity as a function of the penetration depth ϵ. Figure 1 displays the general results.

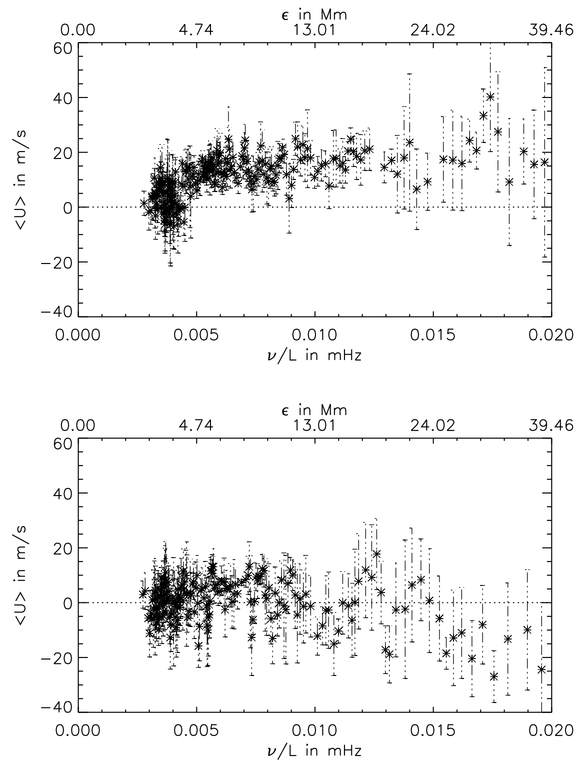

Fig. 1 Estimated averaged horizontal meridional velocity on the northern (top) and southern (bottom) hemisphere as a function of ν/L and as function of mode penetration depth ϵ.

Because of the large scatter in the results we applied a binning of the resulting data points. In this particular case, a binning window of five points has been chosen. This choice reflects a trade-off between a desired rather smooth profile on the one hand and the resolution, which on the other hand should not be too coarse. The reported error bars given in the plots below are due to the statistical scatter of the points within one binning-window. In the plots a positive value of the velocity is equivalent to a poleward flow on the respective hemisphere.

In particular, Fig. 1 (top) gives the averaged horizontal meridional flow on the northern hemisphere. Obviously, various systematic trends are noticeable. First, for modes penetrating between 1–4 Mm the results are distributed around 0 m/s. This feature has not been seen in other investigations of the meridional flow and may correspond to effects of granulation and supergranulation. The true origin of this is, however, not clear. Second, for modes probing deeper below the solar surface there is a clear positive-valued trend of (15 ± 7) m/s. The error bars in this region are relatively small. Third, the observed variation in the curve might be an artefact due to different sensitivities of the p-modes to the flow. For the given mode penetration depths, the results agree well with meridional flow measurements by other local helioseismology techniques like ring-diagram

analysis (Haber et al. (2002), Zaatri et al. (2006)) or time-distance helioseismology (Zhao & Kosovichev (2004)).

Figure 1 (bottom) displays the results for the southern hemisphere. As in the former case, for modes turning close to the solar surface the points are distributed around 0 m/s. For modes turning further down in the deep interior a similar general trend of the curve is observable as on the northern hemisphere. However, the average velocity is about 0 m/s, in contrast to the northern hemisphere where a value of ca. (15 ± 7) m/s is obtained. Additionally the curve shows a far larger amount of scatter, not only expressed in terms of larger error-bars but also in the shape of the curve itself. One reason for this difference between the northern and southern hemisphere might be an error in the determination of the direction of the rotation axis of the Sun (Giles (2000)). Another reason might arise by leakage effects from modes with $m \neq 0$ whose frequencies are shifted by differential rotation. In the following steps we further analyse the measurements for the northern hemisphere data only.

We compare our results obtained for the northern hemisphere with the results presented by Braun & Fan (1998). We analyse a data set set from 1999 whereas Braun & Fan (1998) analysed a data set from 1997. Both results agree well. Differences can only be found on small scales. We considered only modes with an azimuthal order of $m = 0$, whereas the comparable work used a bigger set with $m \neq 0$ and applied an additional averaging. Therefore our errorbars are bigger. Overall, this gives confidence in our newly developed data analysis technique and the resulting measurements.

3.2 Possible extension to greater depths

We are able to measure frequency shifts for modes with harmonic degrees of $l = 73$ – 1013. The modes with the lower harmonic degree penetrate to depths of about 150 Mm. Therefore this method might allow to derive information about the meridional flow throughout 75% of the convection zone. Fig. 2 gives the averaged horizontal meridional velocity $\langle U \rangle$ estimated by this method for modes penetrating to such depths. Between the values of $\epsilon = 20$ Mm and $\epsilon = 100$ Mm there is no clear behaviour visible. Below $\epsilon \approx 110$ Mm the curve shows a trend to become negative-valued. However there are only a few measurements at great depths. Moreover the results are largely scattered which leads to the respective large error-bars. Nevertheless, the measured frequency shifts contain information on the meridional flow down to approximately 150 Mm.

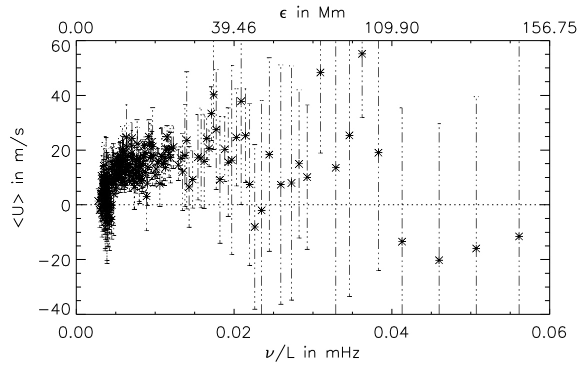

Fig. 2 Estimated integrated horizontal meridional velocity on the northern hemisphere of the Sun as a function of ν/L and mode penetration depth ϵ.

Based on our results we are able to suggest improvements for this technique which we discuss in the following section.

4 Discussion and Future Work

We applied a Fourier-Hankel decomposition to MDI Dopplergrams as described by Braun & Fan (1998) to estimate the average effect of the meridional flow on solar p-modes. We find an average poleward meridional flow velocity on the northern hemisphere in the order of 15 m/s which affects modes penetrating in a depth range of 4–20 Mm. Occurring advection effects on modes penetrating to shallow layers ($\epsilon \approx$ 1–4 Mm) are interpreted as the effect of (super-) granulation. However the latter feature was not found in other helioseismic investigations; its true origin needs to be clarified. Furthermore frequency shifts for modes with an inner turning point at about 150 Mm were measured.

It is encouraging that for modes turning in the upper 20 Mm of the convection zone the inferred average meridional flow agrees well with other independent measurements of the meridional flow velocity (Braun & Fan (1998), Giles (2000), Haber et al. (2002), Zhao & Kosovichev (2004), Zaatri et al. (2006)). Furthermore, as there is an almost perfect agreement between our results and the results of Braun & Fan (1998) we are sure that we were able to rebuilt their original concept.

The clear differences between the northern and southern hemispheres could be either due to unresolved asymmetries in the observational setup at the given observing time or due to other systematic errors. At least for MDI data an offset of typically about 10 m/s in measurements of the meridional flow between the two hemispheres occur, independent from the method of investigation (see e.g. Braun & Fan (1998) or Zaatri et al. (2006)).

The presented method is only a rough first-step-estimation of the profile of the meridional circulation. Future investigations using the decomposition of the wavefield

in space and time and aiming at higher accuracy in the result should take into account the spherical symmetry of the problem. Using Legendre functions instead of Hankel functions will be a first but vital step. Additionally the effect of leakage, potentially not only based on a finite set of degrees (c.f. Gizon & Birch (2005)) but also on the neglected influence of modes with $m \neq 0$, has to be analysed.

We conclude that the presented method – as is – provides a tool for an easy-to-handle analysis of the average solar meridional flow profile in shallow layers. Using longer time series, this method might allow a better frequency resolution to obtain better estimates for the frequency shifts. Moreover, there is some freedom in choosing the investigated range over polar angle. This might allow to assess estimates for the average meridional flow as a function of latitude, too.

The used assumptions break down for modes penetrating to layers below 20 Mm. Moreover the scatter in the data is high at great penetration depths. Nevertheless, we find a negative-value trend in the estimation of the average meridional flow for p-modes probing deeper than 110 Mm. Such a negative value might give a hint to an equatorwards directed meridional flow in the layers above. However, before concluding on the meridional return flow all entrapments in the data analysis need to be resolved.

Most important, a proper inversion of the measured frequency shifts needs to be carried out to estimate the velocity profile over a certain depth range. This will require to deal with the forward problem properly. A normal mode approach needs to be carried out to obtain not only a proper estimation of the effect of all three spatial components of the meridional flow on the mode frequencies but also to obtain integral kernels. Then it will be interesting to see, whether enough data and information could be collected to construct averaging kernels at greater depths. If all these suggested improvements are incorporated, then the meridional flow could be studied in better detail as function of depth, latitude and time with this method.

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